BUDHA DAL PUBLIC SCHOOL PATIALA

SECOND TERM EXAMINATION (16 December 2024)

Class – XII

Paper-Mathematics (Set-A)

Time: 3hrs. M.M. 80

General Instructions:

1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. The anti-derative of $3x^2 + 4x^3$ is

a) $6x + 12x^2$ b) $2x + 3x$ c) $\frac{3x^3}{3} + \frac{4x^4}{8}$ d) $x^3 + x^4$

2. The value of $\int \frac{dx}{sin^2x \cdot cos^2x}$ is equal to

a) $\tan x + \cot x + c$ b) $\tan x - \cot x + c$ c) $\tan x \cot x + c$ d) $\tan x - \cot 2x + c$

3. $\int \frac{e^x(1+x)}{cos^2(e^x x)}\, dx$ is equal to

a) $-\cot\,(e\,x^x) + c$ b) $\tan\,(x\,e^x) + c$ c) $\tan\,(e^x) + c$ d) $\cot\,(x\,e^x) + c$

4. $\int_0^1 \frac{tan^{-1}x}{1+x^2}\, dx$ is equal to

a) $\frac{\pi}{32}$ b) $\frac{\pi^2}{32}$ c) $\frac{2\pi}{33}$ d) $\frac{1}{32}$

5. The value of $\int_0^{\frac{\pi}{2}} \log\left(\frac{4+3\sin x}{4+3\cos x}\right)$ is

a) 2 b) $\frac{3}{4}$ c) 0 d) -2

6. Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the line $x = 0$ and $x = 2$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

7. The order of the differential equation $2x^2 \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is

a) 2 b) 1 c) 0 d) not defined

8. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is

a) $e^x + e^{-y} = c$ b) $e^x + e^y = c$ c) $e^{-x} + e^y = c$ d) $e^{-x} + e^{-y} = c$

9. The integrating factor of the differential equation $x\frac{dy}{dx} + 2y = x^2 (x \neq 0)$ is

a) $2\log x$ b) $\log x$ c) x^2 d) $\frac{1}{x^2}$

10. The unit vector in the direction of vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ is

a) $\frac{1}{6}(\hat{\imath} + \hat{\jmath} + \hat{k})$ b) $\frac{1}{6}(2\hat{\imath} + 3\hat{\jmath} + \hat{k})$ c) $\frac{2\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{14}}$ d) $\frac{-2\hat{\imath}-3\hat{\jmath}+\hat{k}}{\sqrt{14}}$

11. The projection of $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$ is

a) $\frac{10}{\sqrt{17}}$ b) $\frac{5}{3}\sqrt{6}$ c) $5\sqrt{6}$ d) $\frac{10}{\sqrt{5}}$

12. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$, when θ is equal to

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π

13. The direction cosines of a given line are $\frac{1}{K}, \frac{1}{K}, \frac{1}{K}$ then the value of K is

a) $\frac{1}{\sqrt{2}}$ b) $\pm\frac{1}{\sqrt{3}}$ c) 1 d) $\pm\sqrt{3}$

14. The direction ratios of the line $3x + 1 = 6y - 2 = 1 - z$ are

a) 3, 6, 1 b) 3, 6, −1 c) 2, 1, 6 d) 2, 1, −6

15. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called

a) feasible solutions b) constraints c) optimal solutions d) infeasible solutions

16. The point which lies in the half plane $2x + y - 4 \leq 0$ is

a) (0, 8) b) (1, 1) c) (5, 5) d) (2, 2)

17. If $P(A) = \frac{1}{2}, P(B) = 0, then\ P(A/B)$ is

a) 0 b) $\frac{1}{2}$ c) not defined d) 1

18. If $P(A/B) = 0.3, P(A) = 0.4\ and\ P(B) = 0.8$ then P (B/ A) is equal to

a) 0.6 b) 0.3 c) 0.006 d) 0.4

The following questions consists of two statements – Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A.
c) A is true but R is false.
d) A is false but R is true.

A-2

19. Assertion (A) : Fixed point through which line $\frac{x+1}{1} = \frac{y-1}{1} = \frac{z}{4}$ passes is $(-1, 1, 0)$
Reason (R) : Fixed point through which line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ passes is (x_1, y_1, z_1)

20. Assertion (A) : If $P(A) = 0.2$, $P(B) = 0.3$, A and B are independent events, then $P(A \cap B) = 0.06$
Reason (R) : When A and B are independent events then $P(A \cap B) = P(A)/P(B)$

Section - B

21. Find $\int \frac{(x+1)(x+\log x)^2}{x} dx$

22. Find a particular solution satisfying the given condition :

$\cos\left(\frac{dy}{dx}\right) = a \ (a \in R);\ y = 1$ when $x = 0$

23. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}, \vec{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$, show that $(\vec{a} + \vec{b})$ *and* $(\vec{a} - \vec{b})$ are perpendicular to each other.

24. Find the vector equation for the line passing through the points $(-1, 0, 2)$ *and* $(3, 4, 6)$

25. A family has two children. What is the probability that both the children are boys given that atleast one of them is a boy?

Section - C

26. Using properties of definite integral, evaluate $\int_{-5}^{5} |x + 2| dx$

27. Find the general solution of the differential equation $\frac{dy}{dx} - y = \cos x$

28. Find a unit vector perpendicular to each of the vector $(\vec{a} + \vec{b})$ *and* $(\vec{a} - \vec{b})$, where

$\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}, \vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

29. Find the angle between the pair of lines $\vec{r} = 2\hat{\imath} - 5\hat{\jmath} + \hat{k} + \lambda(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$ and

$\vec{r} = 7\hat{\imath} - 6\hat{k} + \mu(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$

30. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

a) Both are red balls

b) One of them is black and other is red

31. Evaluate $\int \frac{x+3}{\sqrt{5-4x+x^2}}$

Section - D

32. Integrate : $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx,\ x \in [0, 1]$

33. Show that the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it

34. Integrate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$, using properties of integrals.

A-3

35. Show that maximum of Z occurs at more than two points: Solve graphically minimize and maximize $Z = 5x + 10y$ subject to the constraints

$$x + 2y \geq 100$$
$$2x - y \leq 0$$
$$2x + y \leq 200$$
$$x, y \geq 0$$

Section – E [Case-study based questions)

36. Read the following and answer the questions.

During examinations a candidate has to reach examination centre and he has three options, i.e. going by bus or scooter or by car. The probability of his going by bus or scooter or by car are $\frac{3}{10}, \frac{1}{10}, \frac{3}{5}$ respectively. The probabilities that he will be late are $\frac{1}{4}$ *and* $\frac{1}{3}$ if he travels by bus and scooter respectively but if he travels by car he is not late.

a) What is the probability of reaching late if he travels by car? (2)

b) What is the probability that candidate reaches late? (2)

37. Read the following and answer the questions.

Employees in an office are following social distancing and during lunch they are sitting at places marked by points A, B and C. Each one is representing position as

$A\,(\hat{\imath} - 2\hat{\jmath} + 4\hat{k}), B\,(5\hat{\imath} + 2\hat{k}), C\,(3\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$

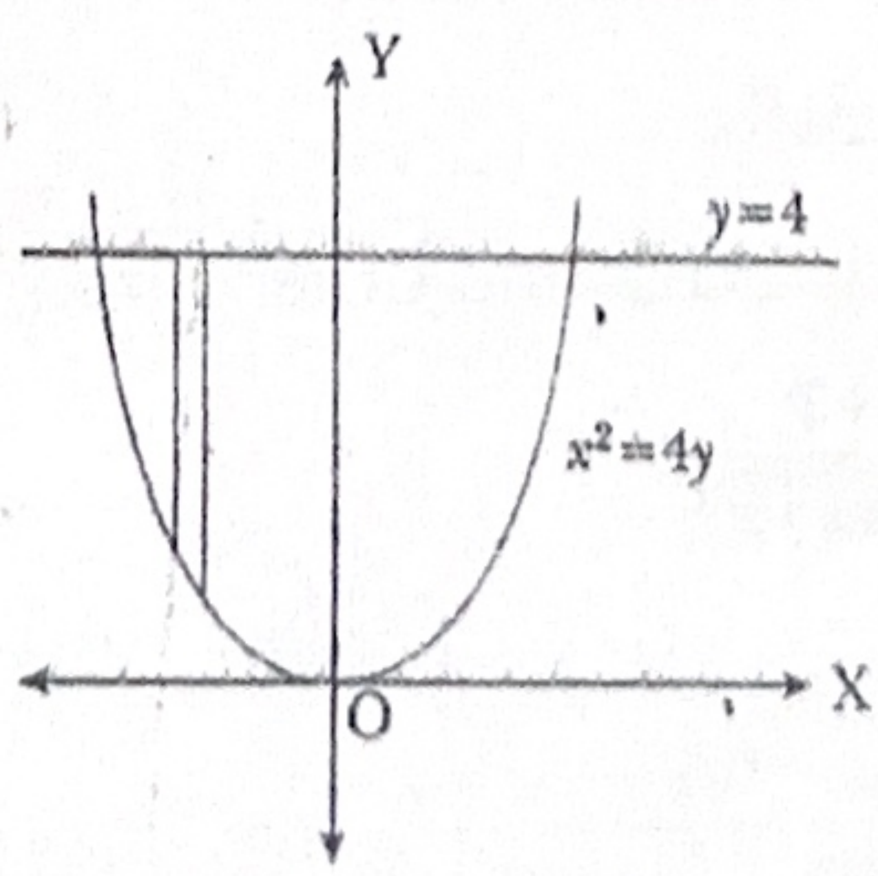
a) Find the distance between B and C. (1)

b) Find the equation of line BC. (CARTESIAN Equation) (1)

c) Find the area of a triangle with vertices A, B and C (2)

38. Read the following and answer the questions.

A student designs an open air Honeybee nest on the branch of a tree, whose plane figure parabolic and the branch of tree is given by a straight line.



(i) Find point of intersection of the parabola and straight line. (1)

(ii) Find the area of each vertical strip. (1)

(iii) Find the area of region bounded by parabola $x^2 = 4y$ and line $y = 4$ (in square units). (2)

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SECOND TERM EXAMINATION (16 December 2024)

Class – XII

Paper-Mathematics (Set-B)

Time: 3hrs. **M.M. 80**

General Instructions:

1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
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Section - A

1. The distance of a point (2, 5, 7) from the $x-axis$ is

   a) 2 b) $\sqrt{74}$ c) $\sqrt{29}$ d) $\sqrt{53}$

2. If α, β, r are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is not true?

   a) $cos^2\alpha + cos^2\beta + cos^2\gamma = 1$

   b) $sin^2\alpha + sin^2\beta + sin^2\gamma = 2$

   c) $cos^2\alpha + cos^2\beta + cos^2\gamma = -1$

   d) $\cos\alpha + \cos\beta + \cos\gamma = 1$

3. For a given LPP, corner points of a closed feasible region are A (3, 5), B (4, 2), C (3, 0) and O (0, 0), then objective function $z = px + qy$ attains maximum at

   a) A b) B c) C or O d) it depends upon values of p & q and points A, B, C

4. The solution set of the inequation $3x + 2y > 3$ is

   a) half plane containing the origin

   b) half plane not containing the origin

   c) the point being on the line $3x + 2y = 3$

   d) none of these

5. The anti derivative of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is

   a) $\frac{1}{3}x^{1/3} + 2x^{1/2} + c$ b) $\frac{2}{3}x^{2/3} + \frac{1}{2}x^2 + c$ c) $\frac{2}{3}x^{3/2} + 2x^{1/2} + c$ d) $\frac{3}{2}x^{3/2} + \frac{1}{2}x^{1/2} + c$

6. $\int \frac{10x^9 + 10^x \log 10 \, dx}{x^{10} + 10^x}$ equals to

   a) $10^x - x^{10} + c$ b) $10^x + x^{10} + c$ c) $(10^x - x^{10})^{-1} + c$ d) $log(10^x - x^{10}) + c$

7. If A and B are events such that $P(A/B) = P(B/A)$

   a) $A \subset B$ but $A \neq B$ b) $A = B$ c) $A \cap B = \emptyset$ d) P (A) = P (B)

B-1

8. Two events A and B are independent if

a) A and B are mutually exclusive
b) $P(A) = P(B)$
c) $P(A) + P(B) = 1$
d) $P(A'B') = [1 - P(A)][1 - P(B)]$

9. Degree of differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) = -1$ is

a) 3 b) 2 c) 1 d) not defined

10. Value of $\int_{1/3}^{1} \frac{(x-x^3)^{1/3}}{x^4}\, dx$ is

a) 6 b) 0 c) 3 d) 4

11. If a is non-zero vector of magnitude 'a' and λ a non-zero scalar, then $\lambda \vec{a}$ is unit vector if

a) $\lambda = 1$ b) $\lambda = -1$ c) $a = |\lambda|$ d) $a = \frac{1}{|\lambda|}$

12. The integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$ is

a) e^{-x} b) e^{-y} c) $\frac{1}{x}$ d) x

13. Let the vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}| = 3$ and $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector, if the angle between $\vec{a}$ and $\vec{b}$

a) $\frac{\pi}{6}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

14. Value of $\int_{-\pi/2}^{\pi/2} (x^3 + x\cos x + \tan^5 x + 1)\, dx$ is

a) 0 b) 2 c) π d) 1

15. $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cdot \cos^2 x}\, dx$ is equal to

a) $\tan x + \cot x + c$ b) $\tan x + \operatorname{cosec} x + c$ c) $-\tan x + \cot x + c$ d) $\tan x + \sec x + c$

16. The number of arbitrary constants in the general solution of a differential equation of 4th order are

a) 0 b) 2 c) 3 d) 4

17. If A and B are any two events such that $P(A) + P(B) - P(A \text{ and } B) = P(A)$, then

a) $P(B/A) = 1$ b) $P(A/B) = 1$ c) $P(B/A) = 0$ d) $P(A/B) = 0$

B-2

18. Let $\vec{a}$ and $\vec{b}$ be any two unit vector and θ is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

a) $\theta = \frac{\pi}{4}$ b) $\theta = \frac{\pi}{3}$ c) $\theta = \frac{\pi}{2}$ d) $\theta = \frac{2\pi}{3}$

The following questions consists of two statements - Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A.
c) A is true but R is false.
d) A is false but R is true.

19. Assertion (A) : Area bounded by the curve $y = f(x)$, *the* $x-axis$ and between $x = a$ *and* $y = b$ is $\int_a^b f(x)dx$

Reason (R) : For area bounded by curve and given ordinates we use y in terms of x

20. Assertion (A) : $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ can be direction cosines of a line

Reason (R) : Three numbers a, b, c are direction cosines of a line if $a^2+b^2+c^2=1$

Section - B

21. Evaluate $\int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8} dx$

22. Find a particular solution satisfying the given condition :

$x(x^2-1)\frac{dy}{dx} = 1$, when $y = 0, x = 2$

23. If $\vec{a} = 5\hat{\imath} - \hat{\jmath} - 3\hat{k}$ *and* $\vec{b} = \hat{\imath} + 3\hat{\jmath} - 5\hat{k}$, then show that vectors $(\vec{a}+\vec{b})$ *and* $(\vec{a}-\vec{b})$ are perpendicular.

24. Find the vector and Cartesian equations of the line passing through the points $(3,-2,-5)$ *and* $(3,-2,6)$

25. Mother, father and son line up at random for a family picture. Find probability when E : son on one end F: father in middle

Section - C

26. Evaluate using property of definite integrals $\int_2^8 |x-5|\, dx$

27. Find the general solution of the differential equation $\frac{dy}{dx} + 2y = \sin x$

28. Find a unit vector perpendicular to each of the vector $\vec{a}+\vec{b}$ *and* $\vec{a}-\vec{b}$, where $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ *and* $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$

29. Find the angle between the pair of lines $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$

30. Problem of solving specific problem independently by A and B are $\frac{1}{2}$ *and* $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

a) problem is solved b) exactly one of them solves the problem

B-3

31. Evaluate $\int \frac{x+2}{2x^2+6x+5}\,dx$

Section – D

32. Evaluate $\int \frac{\sqrt{x^2+1}\,[\log(x^2+1)-2\log x]}{x^4}\,dx$

33. Integrate : $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}$ using properties of integrats.

34. For differential equation $x^2dy + (xy + y^2)dx = 0$, find particular solution

for $y = 1$ *when* $x = 1$

35. Show that maximum Z occurs at more than two points for maximize $Z = x + y$ subject to

$x - y \le -1$
$-x + y \le 0$
$x,\ y \ge 0$

Section – E (Case Study Questions)

36. Read the following and answer the questions.

Often it is taken that a truthful person commands, more respect in the society. A man is known to speak the truth 4 out of 5 times. He throws a die and reports that it is actually a six.

a) What is the probability that man does not speak truth? (2)

b) What is the probability that it is actually a six? (2)

37. Read the following and answer the questions.

Employees in an office are following social distancing and during lunch they are sitting at places marked by points A, B and C. Each one is representing position as

$A\,(\hat{\imath} - 2\hat{\jmath} + 4\hat{k}), B\,(5\hat{\imath} + 2\hat{k}), C\,(3\hat{\imath} + 2\hat{\jmath} + 4\hat{k})$

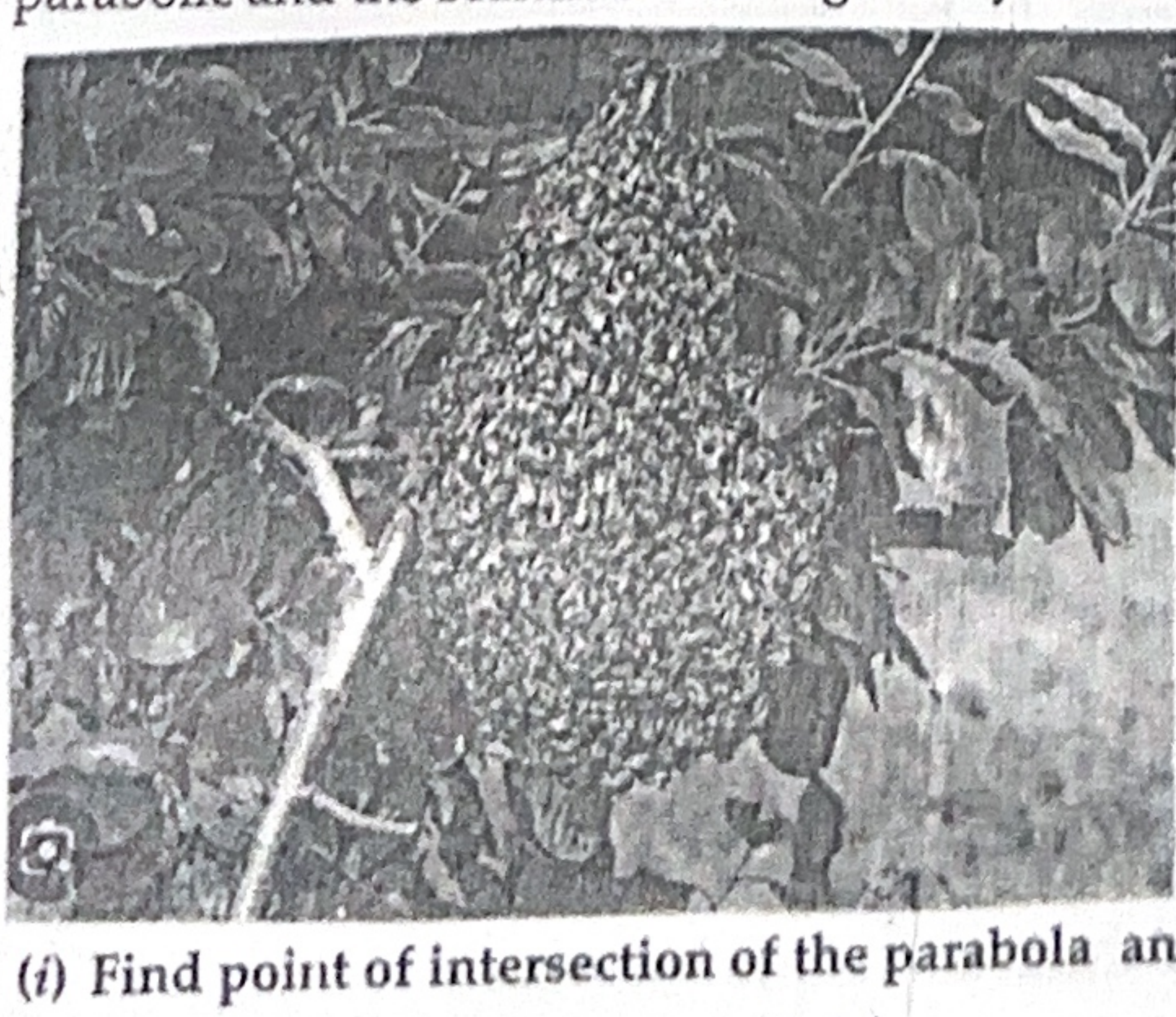
a) Find the distance between A and B. (1)

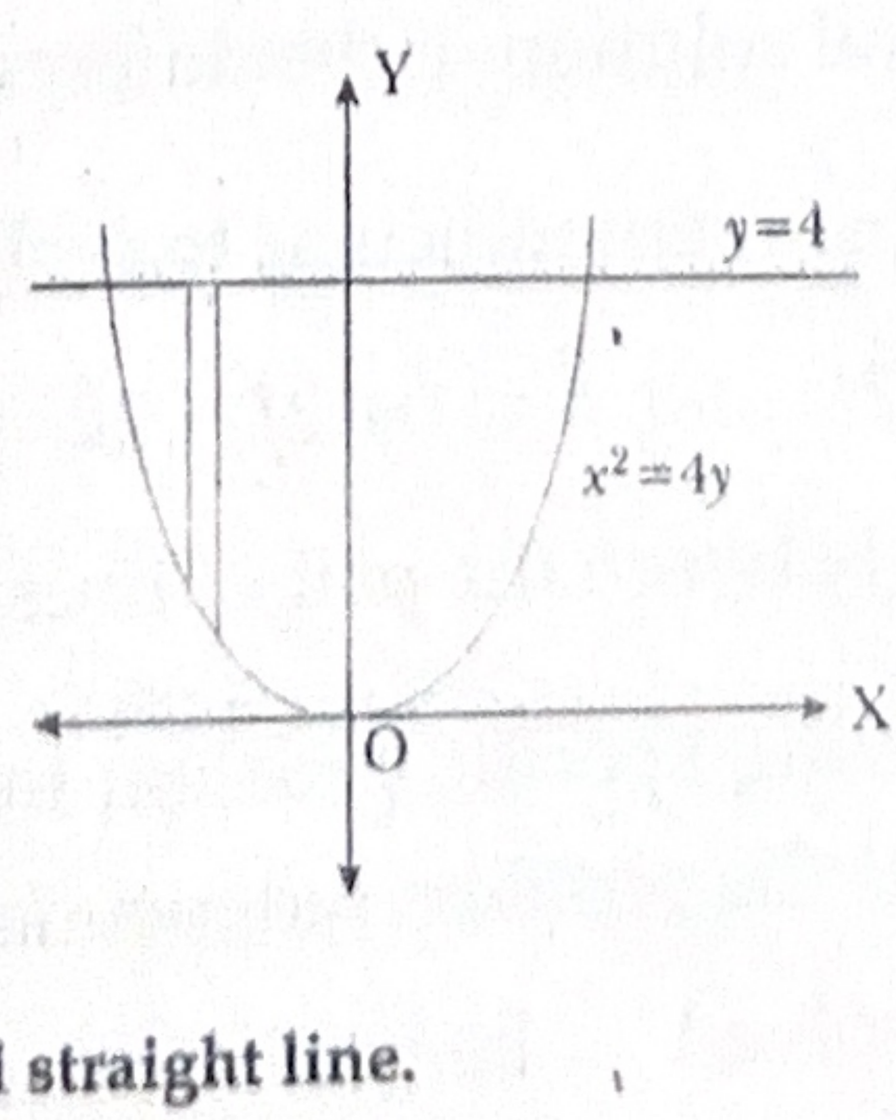
b) Find the Cartesian equation of line AB. (1)

c) Find the area of a triangle with vertices A, B and C (2)

38. Read the following and answer the questions.

A student designs an open air Honeybee nest on the branch of a tree, whose plane figure parabolic and the branch of tree is given by a straight line.





(*i*) Find point of intersection of the parabola and straight line.

(*ii*) Find the area of each vertical strip.

(*iii*) Find the area of region bounded by parabola $x^2 = 4y$ and line $y = 4$ (in square units).