BUDHA DAL PUBLIC SCHOOL PATIALA
FIRST TERM EXAMINATION (16 September 2024)
Class – XII
Paper- Mathematics (Set-A)

Time: 3hrs. **M.M. 80**

General Instructions:

1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. Let R be the relation in the set N given by $R = \{(a, b); a = b - 2, b > 6\}$. Then, the correct option is

   a) $(2,4) \in R$ b) $(3,8) \in R$ c) $(6,8) \in R$ d) $(8,7) \in R$

2. Let R be a relation from R to R on the set of real number defined by

   $R = \{(x, y); x, y \in R \text{ and } x - y + \sqrt{3} \text{ is an irrational number}\}$ Then R is

   a) Reflexive b) transitive c) symmetric d) An equivalence relation

3. If $A = \{1, 2, 3\}, B = \{4,5,7,6\}, then\ f = \{(1,4), (2,5)(3,6)\}$ is

   a) one-one b) many one c) onto d) one-one and onto

4. The Principal value of $cosec^{-1}(2)$ is

   a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

5. The Principal value branch of $cos^{-1}x$ is

   a) $(0, \pi)$ b) $[0, \pi]$ c) $(0, \pi) - \frac{\pi}{2}$ d) $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

6. $\cos\left[\frac{\pi}{3} + cos^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

   a) $\frac{1}{2}$ b) $-\frac{1}{2}$ c) 0 d) -1

7. $tan^{-1}(\sqrt{3}) - sec^{-1}(-2)$ is equal to

   a) π b) $-\frac{\pi}{3}$ c) $\frac{\pi}{3}$ d) $\frac{2\pi}{3}$

8. The number of all possible matrices of order 2 × 3 with each entry 1 or 2 is

   a) 16 b) 64 c) 512 d) 36

9. The matrix $\begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

   a) diagonal matrix b) symmetric matrix c) skew-symmetric matrix d) scalar matrix

10. Given that matrices A and B are of order $3 \times n \text{ and } m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is

    a) $3 \times 5 \text{ and } m = n$ b) 3×5 c) 3×3 d) 5×5

11. If A is a skew symmetric matrix of order 3, then value of $|A|$ is

a) 3 b) 9 c) 27 d) 0

12. If A is any square matrix or order 3×3 such that $|A| = 3$, then the value of $|adj\ A|$ is

a) 3 b) $\frac{1}{3}$ c) 9 d) 27

13. The value of k for which $f(x) = \begin{cases} 3x + 5, x \geq 2 \\ kx^2, x < 2 \end{cases}$ is a continuous function, is

a) $-\frac{11}{4}$ b) $\frac{4}{11}$ c) 11 d) $\frac{11}{4}$

14. The function $f(x) = \begin{cases} x^3, if\ x \leq 2 \\ 10 - x, if\ x > 2 \end{cases}$ is

a) not differentiable at $x = 2$ b) differentiable at $x = 2$

c) not continuous at $x = 2$ d) neither continuous nor differentiable at $x = 2$

15. The rate of the change of area of a circle w.r.t. to its radius $r = 6cm$ is

a) 10π b) 12π c) 8π d) 11π

16. The maximum value of $Z = 3x + 4y$, subject to constiants $x \geq 0, y \geq 0\ and\ x + y \leq 1$ is

a) 3 b) 4 c) 7 d) 10

17. The value of p for which the vectors $2\hat{\imath} + p\hat{\jmath} + \hat{k}\ and - 4\hat{\imath} - 6\hat{\jmath} + 26\hat{k}$ are perpendicular each other, is

a) 3 b) -3 c) $-\frac{17}{3}$ d) $\frac{17}{3}$

18. The value of $(\hat{\imath} \times \hat{\jmath}).\hat{\jmath} + (\hat{\jmath} \times \hat{\imath}).\hat{k}$ is

a) 2 b) 0 c) 1 d) -1

Assertion & Reasoning Questions

The following questions consists of two statements – Assertion (A) and Reason (R). Answe the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for R.
b) Both A and R are true but R is not correct explanation for R.
c) A is true but R is false.
d) A is false but R is true.

19. Assertion (A) : A matrix $A = [1\ 2\ 0\ 3]$ is a row matrix of order 1×4
Reason (R) : A matrix having one row and any number of column is called a row matrix.

20. Assertion (A) : The position vector of a point say $P\ (x, y, z)$ is $\overrightarrow{OP} = \vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and its magnitude is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Reason (R) : if $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then coefficient of $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$ in $\vec{r}\ i.e.\ x, y, z$ are called the direction ratios of vector $\vec{r}$.

Section - B

21. Check the injectivity and surjectivity of function $f: N \to N$ given by $f(x) = 2x + 3$

22. Draw the graph of $f(x) = sin^{-1}x$ in its principal value branch.

23. Find the values of a, b, c and d $\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$

24. Evaluate for x, if $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix} = 0, x \in R$

25. Differentiate $Sin^2(\sqrt{x})$ w.r.t. x

Section - C

26. Find x if $[x \quad -5 \quad 1]\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

27. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$ hence find A^{-1}

28. Discuss the continuity of function $f(x) = \begin{cases} -2, & if\ x \le -1 \\ 2x, & if\ -1 < x \le 1 \\ 2, & if\ x > 1 \end{cases}$

29. Differentiate $x^{\sin x} + 2^{\cos x}$ w.r.t. x

30. Final $\frac{dy}{dx}$, if $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$

31. Prove that $y = \frac{4 \sin \theta}{(2+\cos \theta)} - \theta$, is an increasing function of θ in $\left[0, \frac{\pi}{2}\right]$

Section - D

32. Let L be the set of all lines in XY plane and R be the relation in L defined as

$R = \{(L_1 L_2) : L_1 \parallel L_2\}$, show that R is an equivalence relation. Find the set of all lines related to $y = 2x + 4$

33. Solve the following system of equations using matrix method

$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 2, \quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 5, \quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = -4$

34. Show that right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

35. Maximize $Z = 5x + 3y$

Subjects to $3x + 5y \le 15, \ 5x + 2y \le 10, \ x \ge 0, \ y \ge 0$

Section - E

Case Study Questions

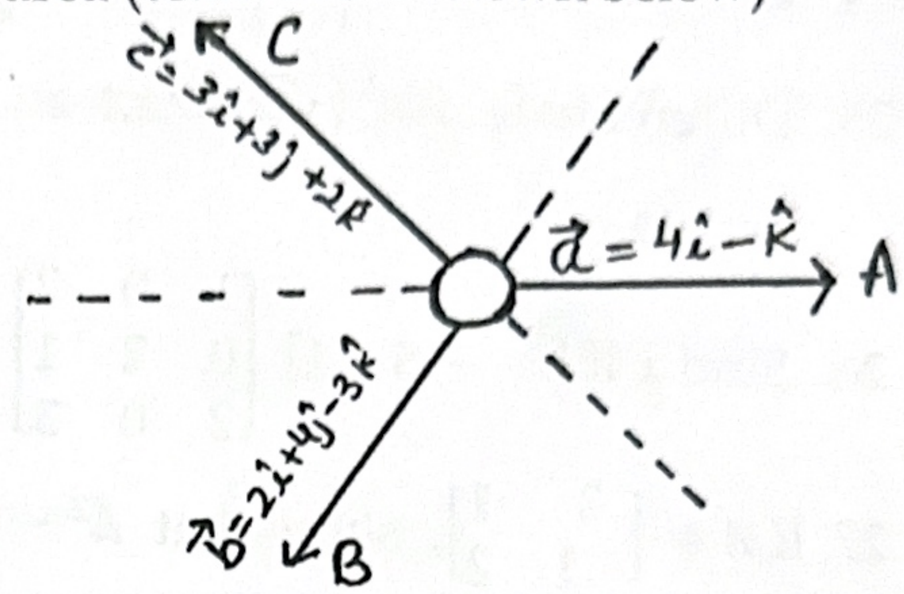
36. **Based on above information, answer these following questions:**

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area (team areas shown below)

Team A pulls along vector $\vec{a} = 4\hat{\imath} - \hat{k}$

Team B pulls along vector $\vec{b} = 2\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$

Team C pulls along vector $\vec{c} = 3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$



1. What are direction cosines of line along which team A pulls? (2)
2. What is the magnitude of the teams taken together? (2)

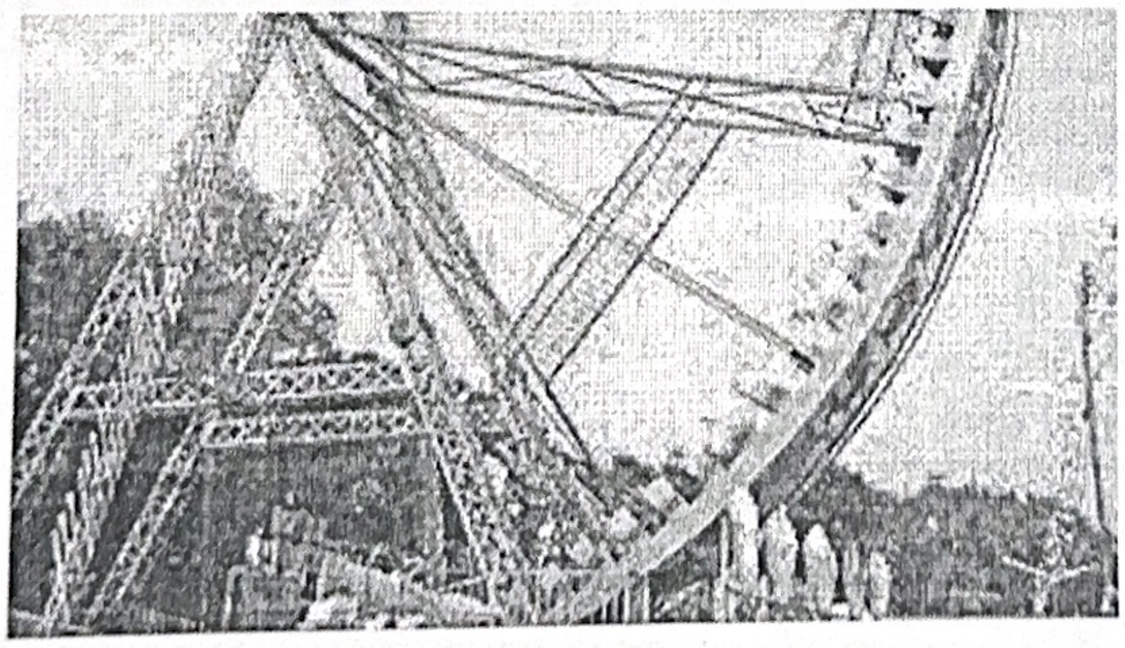
37. Amit, Biraj and Chirag were given the task of creating a square matrix of order 2. Below are the matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively.

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$$

If $a = 4$ *and* $b = -2$, based on the above information answer the following :

1. Find the sum of the matrices A, B and C i.e. A + B + C (1)
2. Find $(bA)^T$ (1)
3. Find $(a + b)\,B + A^T$ (2)

38. Raji visited the amusement park along with her family. The amusement park had a huge swing, which attracted many children. Raji found that the swing traced the path of a parabola as given by $y = x^2$



Answer the following questions using the above information:

1. If $f: R \to R$ be defined by $f(x) = x^2$, then check wither f is an injective function or not. (1)
2. If $f: N \to N$ be defined by $f(x) = x^2$, then check wither f is an injective function or not. (1)
3. Let $f: \{1, 2, 3, \ldots\} \to \{1, 4, 9, \ldots\}$ be defined by $f(x) = x^2$. Check whether the function 'f' is bijective or not by giving suitable reason. (2)

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5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. Let R be the relation in the set N given by $R = \{(a,b); a > 5 - b\}$. Then, the correct option is

   a) (2,3) b) (3,2) c) (3,3) d) (2,1)

2. Relation R in the set A of human beings in a town at particular time given by

   $R = \{(x,y): x \text{ and } y \text{ work at the same place. Then R is}$

   a) Reflexive b) symmetric c) transitive d) An equivalence relation

3. If $A = \{a,b,c\}, B = \{x,y,z\}$ and $g = \{(a,x),(b,y),(c,x)\}$ then g is

   a) one-one b) many one c) onto d) one-one and onto

4. The Principal value of $cos^{-1}(\frac{\sqrt{3}}{2})$ is

   a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

5. The Principal value branch of $cosec^{-1}x$ is

   a) $(-\frac{\pi}{2}, \frac{\pi}{2})$ b) $(-\frac{\pi}{2}, \frac{\pi}{2}) - \{0\}$ c) $[-\frac{\pi}{2}, \frac{\pi}{2}] - \{0\}$ d) $(0,\pi) - \{+\frac{\pi}{2}\}$

6. $\sin[\frac{\pi}{3} + \sin^{-1}(\frac{1}{2})]$ is equal to

   a) 1 b) $\frac{1}{2}$ c) $\frac{1}{3}$ d) $\frac{1}{4}$

7. $\tan^{-1}(1) + \cos^{-1}(-\frac{1}{2})$ is equal to

   a) $\frac{12\pi}{11}$ b) $\frac{11\pi}{12}$ c) $-\frac{\pi}{12}$ d) $\frac{7\pi}{12}$

8. The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

   a) 27 b) 18 c) 81 d) 512

9. The matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is a

   a) identity matrix b) symmetric matrix c) skew-symmetric matrix d) none of these

10. Given that matrices A and B are of order $3 \times n$ and $m \times 5$ respectively, then the order of matrix $C = 5A + 3B$ is

    a) 3×5 and $m = n$ b) 3×5 c) 3×3 d) 5×5

11. If A square matrix of order 3, such that $A\ (adj\ A) = 10\ I$, *then* $|adj\ A|$ is equal to

a) 1 b) 10 c) 100 d) 1000

12. If A is 3×3 matrix such that $|A| = 8$, then $|3\ A|$ is

a) 24 b) 216 c) 72 d) 8

13. The value of k for which $f(x) = \begin{cases} kx^2 + 2, & if\ x \leq 2 \\ 3, & x > 2 \end{cases}$ is a continuous function, is

a) 4 b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) $\frac{1}{4}$

14. The function $f(x) = \begin{cases} x^2, if\ x < 1 \\ 2 - x, if\ x \geq 1 \end{cases}$ is

a) differentiable at $x = 1$ b) not differentiable at $x = 1$

c) not continuous at $x = 1$ d) neither continuous nor differentiable at $x = 1$

15. The rate of the change of area of a circle w.r.t. to its radius r *at* $r = 4cm$ is

a) 4π b) 8π c) 12π d) 2π

16. The minimum value of $Z = 4x + 3y$, subject to constiants $x \geq 0, y \geq 0$ *and* $x + y \leq 1$ is

a) 0 b) 4 c) 3 d) -3

17. The value of λ for which the vectors $3\hat{\imath} - 6\hat{\jmath} + \hat{k}$ *and* $+ 2\hat{\imath} - 4\hat{\jmath} + \lambda\hat{k}$ are parallel

a) $\frac{3}{2}$ b) $-\frac{3}{2}$ c) $\frac{2}{3}$ d) $\frac{1}{3}$

18. If $\vec{a}$ *and* $\vec{b}$ are two non-zero vectors such that projection of $\vec{a}$ *on* $\vec{b}$ is 0. The angle between $\vec{a}$ *and* $\vec{b}$

a) $\frac{\pi}{2}$ b) π c) $\frac{\pi}{4}$ d) 0

Assertion & Reasoning Questions

The following questions consists of two statements – Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for R.
b) Both A and R are true but R is not correct explanation for R.
c) A is true but R is false.
d) A is false but R is true.

19. Assertion (A) : If $|A| = 3, then\ |3A| = 27$, if A is a order of 3×3

Reason (R) : if k is a scalar and A is $n \times n$ matrix then $|kA| = k^n|A|$

20. Assertion (A) : If $\vec{r} = 3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ and its magnitude is $|\vec{r}| = \sqrt{17}$

Reason (R) : if $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, then $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$

Section – B

21. Check the injectivity and surjectivity of function $f: R \to R$ given by $f(x) = 2x + 5$

22. Draw the graph of $f(x) = cos^{-1}x$ in its principal value branch

23. Find the values of a, b, c and d $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

24. If $\begin{vmatrix} x+1 & x-1 \\ x-3 & x+2 \end{vmatrix} = \begin{vmatrix} 4 & -1 \\ 1 & 3 \end{vmatrix}$, find x

25. Differentiate $sec(tan\sqrt{x})$ w.r.t. x

Section - C

26. Find x, if $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

27. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$

28. Discuss the continuity of function $f(x) = \begin{cases} 2x, & x < 0 \\ 0, & 0 \le x \le 1 \\ 4x, & x > 1 \end{cases}$

29. Differentiate $(\sin x)^x + sin^{-1}\sqrt{x}$ w.r.t. x

30. Final $\frac{dy}{dx}$, if $x = (\theta - \sin\theta)$, $y = a(1 + \cos\theta)$

31. Prove that $y = \log(1+x) - \frac{2x}{2+x}$, $x > -1$, is an increasing function of x throughout its domain

Section - D

32. Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b): |a - b| \text{ is divisible by } 2\}$ is an equivalence relation. Write the set of elements related to 1.

33. Solve the following system of equations using matrix method
$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$; $\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$, $\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

34. Prove that volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

35. Solve the following LPP graphically, Minimize $Z = -3x + 4y$
Subjects to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$

Section - E

Case Study Questions

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If $a = 4$ and $b = -2$, based on the above information answer the following :

1. Find the sum of the matrices A, B and C i.e. A + B + C (1)
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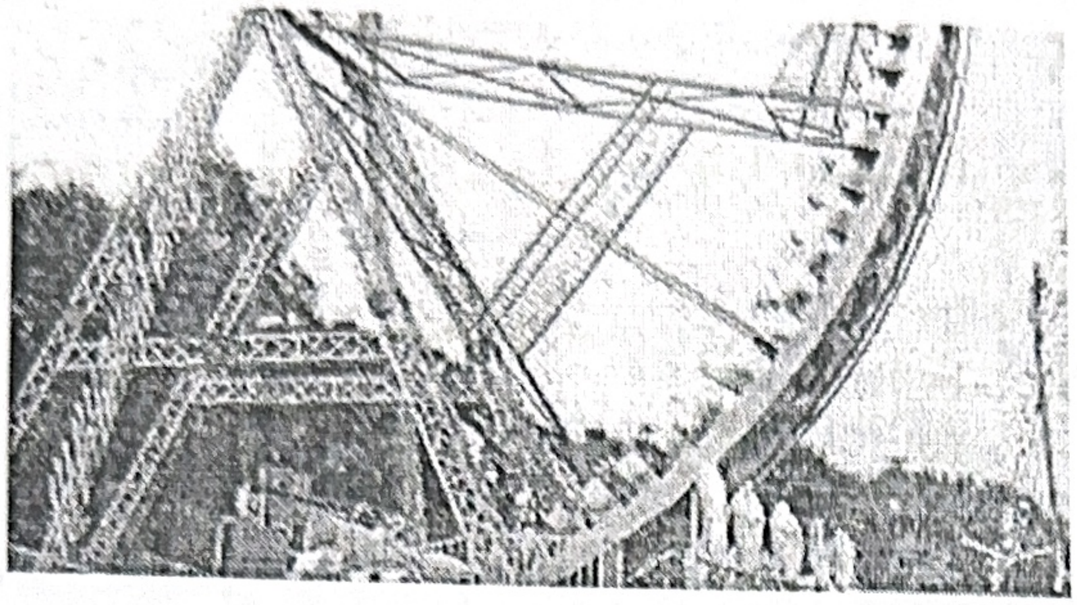

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Team C pulls along vector $\vec{c} = 3\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$

1. What are direction cosines of line along which team A pulls? (2)
2. What is the magnitude of the teams taken together? (2)