BUDHA DAL PUBLIC SCHOOL PATIALA

PRE BOARD EXAMINATION (16 January 2025)

Class – XII

Paper-Mathematics (Set-A)

Time: 3hrs. **M.M. 80**

General Instructions:

1. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. Total number of possible matrices or order 3×2 with each entry 5 or 7

   a) 6 b) 12 c) 64 d) 32

2. Given matrix $A = [a_{ij}]$ of order 3×3, where $a_{ij} = \begin{cases} i + 3j\ , & i < j \\ 5\ , & i = j \\ i - 3j & i > j \end{cases}$

   The number of elements in matrix A which are more than 3 are

   a) 6 b) 3 c) 5 d) 9

3. Vectors $\hat{\imath}, \hat{\jmath}, \hat{k}$ are unit vectors along $x, y\ and\ z - axes$ respectively, then the value of $\hat{\imath}.(\hat{\jmath} \times \hat{k}) - 2\hat{k}.(\hat{\imath} \times \hat{\jmath}) + \hat{\jmath}.(\hat{\imath} \times \hat{k})$ is

   a) -4 b) 0 c) -1 d) -2

4. The function $f(x) = [x], -3 < x < 4, x \in R$, where $[x]$ represents greatest integer $\leq x$ is discontinuous for one of the values of x equal to

   a) 8 b) 1 c) 4 d) 2.5

5. If $\int f(x)dx = \frac{2^x}{log_e 2} + C$, then $f(x)$ is

   a) 2^x b) $\frac{2^x}{log_e 2}$ c) $2^x . \log 2$ d) 2^{x-1}

6. If $p\ and\ q$ are degree and order of differential equation $\frac{dy}{dx} + \frac{1}{dy/dx} = x$, then value of $p - 3q$ is

   a) -5 b) -3 c) -1 d) not defined

7. The point which does not lie in the half plane of $3x - y < 4$ is
   a) (0, 0) b) (1, 1) c) (1, -2) d) (1, 2)

A-1

8. If vector $3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $2\hat{\imath} + p\hat{\jmath} + \frac{2}{3}\hat{k}$ are parallel vectors then the value of p is

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $-\frac{3}{4}$ d) $-\frac{4}{3}$

9. The value of $\int_1^4 \frac{x}{\sqrt{x^2-1}}\,dx$ is

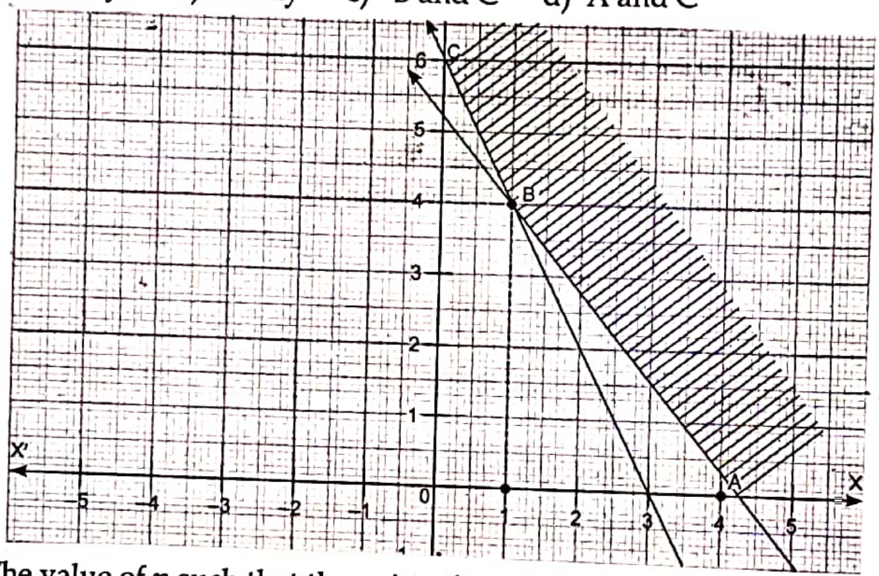
a) $\frac{1}{2}\sqrt{15}$ b) $\sqrt{15}$ c) $2\sqrt{15}$ d) $\sqrt{15} - 1$

10. If A is a square matrix such that $|A| = 7, then\ |A^{-1}|$ is

a) 49 b) $\frac{1}{7}$ c) -7 d) 14

11. For the given unbounded feasible region, minimum of $Z = 3x + 5y$ occurs at

a) A only b) B only c) B and C d) A and C



12. The value of p such that the points $(3,-2), (p,2) and\ (8,8)$ are collinear is

a) 0 b) 4 c) 5 d) 3

13. For two given square matrices A and B, $BA = 9I, then\ A^{-1}B^{-1}$ is

a) $\frac{1}{9}I$ b) $9B^{-1}$ c) $9A^{-1}$ d) $9I$

14. If $A\ and\ B$ are events such that $P\left(\frac{A}{B}\right) = P\left(\frac{B}{A}\right)$, then

a) $A \subset B$ b) $A = B$ c) $A \cap B = \emptyset$ d) $P(A) = P(B)$

15. General solution of differential equation $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ is

a) $sin^{-1}x + sin^{-1}y = C$ b) $sin^{-1}y + cos^{-1}x = C$

c) $tan^{-1}x + tan^{-1}y = C$ d) $tan^{-1}x - tan^{-1}y = C$

A-2

16. If $y = e^{\sin^{-1}x}$, then $(1-x^2)\,y_1^2$ is equal to

a) $\sqrt{1-x^2}$ b) $\sin^{-1}x$ c) y^2 d) y

17. If θ is acute angle between any two non-zero vectors $\vec{a}$ and $\vec{b}$ such that $|\vec{a}\cdot\vec{b}| = |\vec{a}\times\vec{b}|$, then value of θ is

a) $\frac{3\pi}{4}$ b) 0 c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

18. Which triplet can represent direction cosines of a line?

a) 1, 2, 3 b) −1, 1, 1 c) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ d) $\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

In the following questions a statements - Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:
a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A.
c) A is true but R is false.
d) A is false but R is true.

19. Assertion (A) : $\sin^{-1}(1.01)$ exists.

Reason (R) : Domain of $\sin^{-1}x$ is $x \in [-1, 1]$

20. Assertion (A) : The scalar projection of vectors $\hat{\imath} + \hat{\jmath} + 2\hat{k}$ along vector $\hat{\imath}$ is 1.

Reason (R) : The scalar projection of $\vec{a}$ along $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$

Section - B

21. Evaluate $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

OR

Represent $\cot(\sin^{-1}x)$ in terms of x only

22. The circular waves are moving at the rate of 0.7 cm/s. At what rate the area is changing when radius of circular wave is 7cm?

23. Given $\vec{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\vec{b} = p\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$ and $\vec{c} = 5\hat{\imath} - \hat{k}$ find the value of p such that $\vec{a} + 2\vec{c}$ is perpendicular to $\vec{b}$.

OR

Given vector $\vec{a} = 6\hat{\imath} - 8\hat{\jmath} + 12\hat{k}$, write

a) Components of a vector along with $x - axis, y - axis$ and $z - axis$

b) Direction cosines of $\vec{a}$

24. If $y = Ae^{7x} + Be^{-7x}$, prove that $y_2 - 49y = 0$

25. Show that $|\vec{b}|\,\vec{a} + |\vec{a}|\,\vec{b}$ and $|\vec{b}|\,\vec{a} - |\vec{a}|\,\vec{b}$ are orthogonal vectors.

Section – C

26. Evaluate : $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (3x^2 + tan^5x - sin^7x + 9)\, dx$

27. Find the probability of getting a total of 9 in a throw of two dice, if it is known that number 5 occurs on the first dice.

OR

A bag contains 5 red and 3 blue balls. If two balls are drawn at random, find the probability distribution of getting a red ball. Also, find mean of the distribution.

28. Evaluate : $\int_0^{\frac{\pi}{4}} log\,(1 + \tan x)\, dx$

OR

Evaluate : $\int_0^3 (|x| + |x - 2|)\, dx$

29. Solve the differential equation $x \log x \frac{dy}{dx} + y = \frac{2}{x}\, long\, x$

OR

Solve the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$

30. Solve the following LPP graphically :

Maximise $Z = 2x - y + 5$, subject to constraints $3x + 4y \le 60, x + 3y \le 30, x \ge 0, y \ge 0$

31. Evaluate : $\int \frac{1}{1+\tan x}\, dx$

Section – D

32. Using integration, find the area of the region enclosed between the circle $x^2 + y^2 = 16$ and the lines $x = -2$ and $x = 2$

33. If $f : R \to R$ is defined by $f(x) = |x|^3$, show that $f''(x)$ exists for all real x and find it.

OR

If $(x - a)^2 + (y - b)^2 = c^2$, for some $c > 0$, Prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{d^2x}}$ is a constant independent of a and b

34. Find the shortest distance between these two lines

$\vec{r} = (\hat{\imath} - \hat{\jmath}) + \lambda\,(2\hat{\imath} + \hat{k})$ and $\vec{r} = (2\hat{\imath} - \hat{\jmath}) + \mu\,(\hat{\imath} - \hat{\jmath} - \hat{k})$

OR

A-4

If $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, then represent $\vec{b}$ as $\vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is a parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$

35. The equation of the path traced by a roller – coaster is given by
$y = ax^3 + bx^2 + cx - 27$ (where $a, b, c \in R$ and $a \neq 0$) if the roller coaster passes through the points (-1, 0), (-2, 35) and (1, -40), then find the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method.

Section – E

36. Read the following and answer the questions.

An organization conducted bike race under 2 different categories – boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}, G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions.

On the basis of above information, answer the following questions:

a) Ravi wishes to form all the relations possible from B to G. How many such relations are possible? (1)

b) Among these relations, how many are functions from B to G? (1)

c) (i) Find the total number of one-one and onto functions which can be defined from B to G. (2)

OR

c) (b) If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; (where $0 \leq x \leq 20\sqrt{2}$ and $0 \leq y \leq 200$), then state whether the track represents a one-one and onto function or not. (Justify)

37. Read the following passage and answer the questions given below:

The village panchayat decided to build an open rectangular tank to store water for future needs. Depth of the tank 2 m and volume of tank is 8 m³.

The cost of building tank is Rs. 70/m² for the base and Rs. 45/m² for the sides. Taking base dimensions as x m and y m

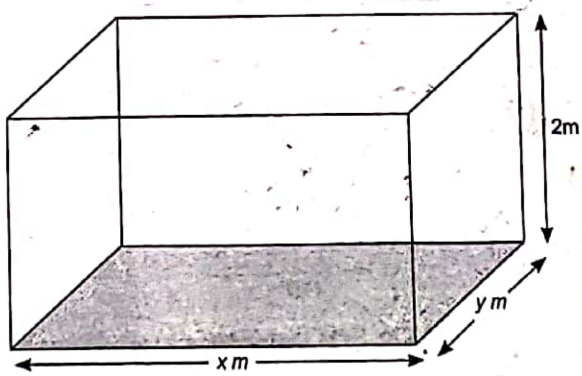
a) Write a relation between x and y (1)

b) Find the cost of building tank in terms of x (1)

c) For what value of x, cost is minimum? (2)

OR

c) Find the marginal cost when $x = 3m$.



A-5

38. **Read the following passage and answer the questions given below:**

In an Office three employees James, Sophia and Oliver process incoming copies of a certain form. James processes 50% of the forms, Sophia processes 20% and Oliver the remaining 30% of the forms. James has an error rate of 0.06, Sophia has an error rate of 0.04 and Oliver has an error rate of 0.03.

(*i*) Find the probability that Sophia processed the form and committed an error. (2)

(*ii*) Find the total probability of committing an error in processing the form. (2)

A - 6

BUDHA DAL PUBLIC SCHOOL PATIALA

PRE BOARD EXAMINATION (16 January 2025)

Class – XII

Paper-Mathematics (Set-B)

Time: 3hrs. **M.M. 80**

General Instructions:

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3. Section B has 5 Very Short Answer type questions of 2 marks each.
4. Section C has 6 Short Answer type questions of 3 marks each.
5. Section D has 4 Long Answer type questions of 5 marks each.
6. Section E has 3 case based studies of 4 marks each.

Section - A

1. If $A = [a_{ij}]$ is a matrix of order 2×3 such that $a_{ij} = \begin{cases} 2 & , \; if\ i+j\ is\ even \\ i-2j, & if\ i+j\ is\ odd \end{cases}$

a) $\begin{bmatrix} 2 & 0 & 2 \\ -3 & 2 & 0 \end{bmatrix}$ b) $\begin{bmatrix} 2 & 0 \\ 2 & -2 \\ 2 & 4 \end{bmatrix}$ c) $\begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 2 & -4 \end{bmatrix}$ d) $\begin{bmatrix} 2 & -3 & 2 \\ 0 & 2 & -4 \end{bmatrix}$

2. If A is a matrix of 3, such that $|ajd\ A| = 5$, which of these could be the value of $|A|$?

a) 5^2 b) 5 c) $\sqrt{5}$ d) $3\sqrt{5}$

3. The point which lies in the half plane of $x + 2y > 5$ is

a) $(0, 0)$ b) $(-1, 1)$ c) $(0, 3)$ d) $(4, -5)$

4. Which of the following statements is false for the function, $f(x) = [x]$, $x \in R$, where $[x]$ represents greatest integer $\leq x$?

a) $f(x)$ is not continuous at $x = 3$

b) $f(x)$ is continuous at $x = 3$

c) $f(x)$ is not differentiable at $x = 3$

d) $f(x)$ is continuous at $x = 3.5$

5. The derivative of 2^x w.r.t 3^x is

a) $\left(\frac{3}{2}\right)^x \frac{\log 2}{\log 3}$ b) $\left(\frac{2}{3}\right)^x \frac{\log 3}{\log 2}$ c) $\left(\frac{2}{3}\right)^x \frac{\log 2}{\log 3}$ d) $\left(\frac{3}{2}\right)^x \frac{\log 3}{\log 2}$

6. If $y = log_2 x^2$, then value of $\frac{dy}{dx}$ $at\ x = e^{-1}$ is

a) $2e$ b) $2e\ log_2 e$ c) $2e\ log_e 2$ d) $\frac{2}{e}$

7. For maximizing $Z = 3x + 2y$ under constraints $x + 2y \leq 10$, $3x + y \leq 15$, $x \geq 0$ and $y \geq 0$, which of the following is not a corner point of feasible region?

a) (0, 5) b) (4, 5) c) (5, 0) d) (0, 0)

B - 1

8. In which of these intervals, the function $f(x) = 3x^2 - 4x$, is strictly decreasing?

a) $(-\infty, 0)$ b) $(0, 2)$ c) $\left(\frac{2}{3}, \infty\right)$ d) $(-\infty, \infty)$

9. The value of k for which the function, $f(x) = \begin{cases} 3x^2, & if\ x > 0 \\ kx, & if\ x < 0 \end{cases}$ is continuous at $x = 0$, is

a) 0 b) 1 c) -1 d) no value of k

10. The diameter of a sphere is $\frac{3}{2}$ $(2x + 3)$, the rate of change of its surface area with respect to x is

a) $18\pi\ (2x + 3)$ b) $\frac{3}{2}$ c) $9\pi\ (2x + 3)$ d) $\frac{3}{4}$

11. The value of $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} x^3 \sin^4 x\ dx$ is

a) 0 b) $\frac{\pi}{2}$ c) π d) $\frac{\pi^2}{4}$

12. $\int \sec^2 (4 - 3x)\ dx$ is equal to

a) $\cos^2 (4 - 3x) + C$ b) $-\frac{1}{3} \tan (4 - 3x) + C$ c) $-\frac{1}{3} \cos^2 (4 - 3x) + C$ d) $-\frac{1}{9} \tan (4 - 3x) + C$

13. A boy is generating a sinusoidal wave in a taut horizontal rope by raising and then lowering one end of rope, represented by $y = \sin x$. Then, area of the region bounded by $y = \sin x$ between $x = 0, x = \frac{3\pi}{2}$ and $x - axis$ is

a) $\frac{3}{2}$ *sq units* b) 2 *sq units* c) $\frac{3}{2}$ *sq units* d) 4 *sq units*

14. In which of the given differential equations is the degree equal to its order?

a) $x\frac{dy}{dx} - \frac{d^2y}{dx^2} = 0$ b) $x^2\frac{dy}{dx} + \tan y - \left(\frac{d^2y}{dx^2}\right)^2 = 0$

b) $\left(\frac{d^3y}{dx^3}\right) + \sin\left(\frac{dy}{dx}\right) = 0$ d) $x\left(\frac{dy}{dx}\right)^3 - \left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} - 7 = 0$

15. For the vector $(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) \times \hat{k}$, component vector along $x - axis$ is

a) $-3\hat{\imath}$ b) -3 c) $2\hat{\jmath}$ d) $4\hat{k}$

16. Area of parallelogram whose one side and one diagonal are represented along the vector $3\hat{\imath}$ and $-5\hat{\jmath}$ is

a) 4 sq units b) 8 sq units c) 15 sq units d) 7.5 sq units

17. A line makes angles 60^0, 60^0 and 45^0 with x-axis, y-axis and z-axis respectively, then direction ratios of line are

a) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$ b) $(\sqrt{2}, 1, \sqrt{2})$ c) $(1, 1, \sqrt{2})$ d) $(\sqrt{2}, \sqrt{2}, 1)$

B-2

18. Line $\vec{r} = 2\hat{j} - 3\hat{k} - \hat{i} + \mu(\hat{j} - 3\hat{k} + 5\hat{i})$ is along the direction vector

a) $2\hat{j} - 3\hat{k} - \hat{i}$ b) $\hat{j} - 3\hat{k} - \hat{i}$ c) $5\hat{j} - 3\hat{k} + \hat{j}$ d) $\hat{i} - 3\hat{k} + 2\hat{j}$

In the following questions a statements - Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for A.
b) Both A and R are true but R is not correct explanation for A.
c) A is true but R is false.
d) A is false but R is true.

19. Assertion (A) : The domain of the function $f(x) = sin^{-1}(3x)$ $is -\frac{1}{3} \leq x \leq \frac{1}{3}$.

Reason (R) : Principal value of $cos^{-1}x$ lies in the internal $[0, \pi]$

20. Assertion (A) : If A and B are two events such that $P(A \cap B) = 0$, then $P\left(\frac{A}{(A \cup B)}\right) = \frac{P(A)}{P(A) \times P(B)}$

Reason (R) : $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$

Section - B

21. The volume of a cube is increasing at a constant rate. Prove that the rate of increase of its surface area varies inversely as length of the side.

OR

Find the maximum profit that a company can make, if the profit function is given by $P(x) = 105 + 48x - x^2$, where x represents the number of units sold and P represents the profit.

22. Evaluate : $\int x(a - x)^{20}\, dx$, where a is a constant.

23. Find the value of $sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + 2\, tan^{-1}(\sqrt{3})$

OR

If $tan^{-1}\left(\frac{x+1}{\sqrt{3}}\right) < \frac{\pi}{3}$, then find the range of values of x

24. If $x = \cot t$ and $y = cosec^2\, t$, then find (i) $\frac{dy}{dx}$ (ii) $\frac{d^2y}{dx^2}$

25. Find the angle between the lines $\vec{r} = \hat{j} - 3\hat{k} - 3\hat{i} + \mu(2\hat{j} - \hat{k} + 4\hat{i})$ and $\frac{x+5}{3} = \frac{2y-2}{1} = \frac{z+1}{-2}$

Section - C

26. $\int \frac{x^2+1}{(x^2+2)(x^2+3)}\, dx$

OR

B-3

$\int \log(1+x^2)\, dx$

27. If $x = a\sin t - b\cos t$ and $y = a\cos t + b\sin t$, then prove that $\frac{d^2y}{dx^2} = -\left(\frac{x^2+y^2}{y^3}\right)$

28. Evaluate : $\int_{-1}^{3} |2x+1|\, dx$

OR

Evaluate : $\int_2^7 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\, dx$

29. The random variable X has the following probability distribution where a and b are some constants:

X	1	2	3	4	5
P(X)	0.2	a	a	0.2	b

If the mean E(X) = 3, then find values of a and b and hence determine $P(X \geq 3)$

30. Solve the differential equation $2ye^{x/y}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$

OR

Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$

31. Solve LPP graphically Min $Z = 3x + 9y$,

subject to constraints are

$x + 3y \leq 60,\ \ x + y \geq 10,\ \ x \leq y,\ \ x \geq 0,\ \ y \geq 0$

Section – D

32. Find the bounded by the x-axis, circle $x^2 + y^2 = 32$ and the line $y = x$ in first quadrant.

33. Find $'a'$ and $'b'$, if the function given by $f(x) = \begin{cases} ax^2 + b, if\ x < 1 \\ 2x + 1, if\ x \geq 1 \end{cases}$ is differentiable at $x = 1$

OR

if $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$

34. Find the image of the point, whose position vector is $2\hat{\imath} - \hat{\jmath} - \hat{k}$, in the line mirror $\vec{r} = (2\hat{\imath} + \hat{\jmath} - 2\hat{k}) + \lambda\,(\hat{\imath} + 3\hat{\jmath} + 2\hat{k})$.

OR

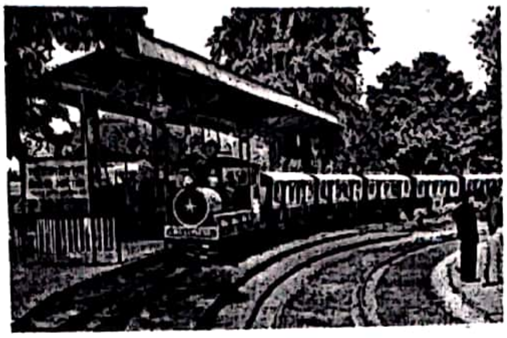
Find the shortest distance the lines $\frac{x-1}{2} = \frac{y+1}{3} = z$ and $\frac{x+1}{5} = \frac{y-2}{1}; z = 2$. Are the lines intersecting?

35. The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; (where $0 \leq x \leq 14$ and $a, b, c \in R$ and $a \neq 0$) with respect to a XY-coordinate system in the vertical plane. The ball passes through the points (2, 15), (4, 25) and (14, 15). Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball.

B-4

Section - E

36. Students of a school are taken to a railway museum to learn about railways heritage and its history.



An exhibit in the museum depicted many rail lines on the track near the railway station. Let L be the set of all rail lines on the railway track and R be the relation on L defined by

$R = \{(l_1, l_2) : l_1 \text{ is parallel to } l_2\}$

On the basis of the above information, answer the following questions :

(i) Find whether the relation R is symmetric or not. (1)

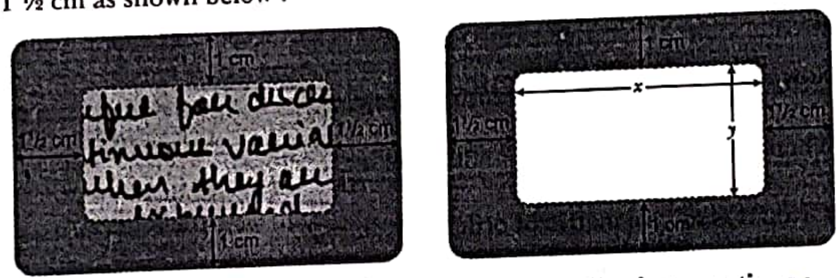
(ii) Find whether the relation R is transitive or not. (1)

(iii) (a) If one of the rail lines on the railway track is represented by the equation $y = 3x + 2$, then find the set of rail lines in R related to it. (2)

OR

(iii) (b) Let S be the relation defined by $S = \{(l_1, l_2) : l_1 \text{ is perpendicular to } l_2\}$ check whether the relation S is symmetric and transitive.

37. A rectangular visiting card is to contain 24 sq.cm. of printed matter. The margins at the top and bottom of the card are to be 1 cm and the margins on the left and right are to be 1 ½ cm as shown below :



On the basis of the above information, answer the following questions :

(i) Write the expression for the area of the visiting card in terms of x. (2)

(ii) Obtain the dimensions of the card of minimum area. (2)

38. A departmental store sends bills to charge its customers once a month. Past experience shows that 70% of its customers pay their first month bill in time. The store also found that the customer who pays the bill in time has the probability of 0.8 of paying in time next month and the customer who doesn't pay in time has the probability of 0.4 of paying in time the next month.

Based on the above information, answer the following questions :

(i) Let E_1 and E_2 respectively denote the event of customer paying or not paying the first month bill in time. (1)

Find $P(E_1)$, $P(E_2)$.

(ii) Let A denotes the event of customer paying second month's bill in time, then find $P(A|E_1)$ and $P(A|E_2)$. (1)

(iii) (a) Find the probability of customer paying second month's bill in time. (2)

OR

(iii) (b) Find the probability of customer paying first month's bill in time if it is found that customer has paid the second month's bill in time.

B-5

BUDHA DAL PUBLIC SCHOOL PATIALA

PRE BOARD EXAMINATION (16 January 2025)

Class – XII

Paper- Applied Mathematics

Time: 3hrs. **M.M. 80**

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Section - A

1. The least non-negative remainder when 3^{15} is divided by 7 is

   a) 1 b) 5 c) 6 d) 7

2. The ratio in which a grocer mixes two varieties of pulses costing Rs. 85 per kg and Rs. 100 per kg respectively so as to get a mixture worth Rs. 92 per kg is

   a) 7:8 b) 8:7 c) 5:7 d) 7:5

3. If $A = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -5 & 10 \\ -10 & -5 \end{bmatrix}$, then AB is

   a) $\begin{bmatrix} -5 & 10 \\ 0 & -5 \end{bmatrix}$ b) $\begin{bmatrix} 0 & -5 \\ 25 & 10 \end{bmatrix}$ c) $\begin{bmatrix} 10 & -25 \\ -5 & 0 \end{bmatrix}$ d) $\begin{bmatrix} -5 & 10 \\ 0 & -25 \end{bmatrix}$

4. If A is a square matrix of order 3 × 3 such that $|A| = 8$, then $|3A|$ is

   a) 8 b) 24 c) 72 d) 216

5. If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A =$

   a) I b) A c) 2A d) 3I

6. If $x = 2at,\ y = at^2$, where a is constant, then $\frac{d^2y}{dx^2}$ at $x = \frac{1}{2}$ is

   a) $\frac{1}{2a}$ b) 1 c) $2a$ d) None of these

7. If the function $f(x) = x + \cos x + b$ is strictly decreasing over R, then

   a) $b < 1$ b) no value of b exists c) $b \leq 1$ d) $b \geq 1$

8. $\int \frac{1}{x + x\log x} dx$ is equal to

   a) $1 + \log x + C$ b) $x + \log x + C$ c) $x(1 + \log x) + C$ d) log (1+log x) +C

1 –

9. If a random variable X has the following probability distribution

X:	0	1	2	3	4	5	6	7
P(X):	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2+K$

Then $P(X \geq 6)$ is

a) $\frac{19}{100}$ b) $\frac{81}{100}$ c) $\frac{9}{100}$ d) $\frac{91}{100}$

10. The mean and variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

a) $\frac{219}{256}$ b) $\frac{128}{256}$ c) $\frac{28}{256}$ d) $\frac{37}{256}$

11. Standard deviation of a sample from a population is called a

a) Standard error b) parameter c) statistic d) central limit

12. For a student's t-test, the test statistic t is given by

a) $t = \bar{x} - u$ b) $t = \frac{\bar{x}}{s}$ c) $t = \frac{\bar{x}-u}{s}$ d) $t = \frac{\bar{x}-u}{s/\sqrt{n-1}}$

13. For the given five values : 35, 70, 36, 59, 64, the three years moving averages are

a) 47, 53, 55 b) 53, 47, 45 c) 47, 55, 53 d) 45, 55, 57

14. At what rate of interest will the present value of a perpetuity of Rs. 300 payable at the end of each quarter be Rs. 24,000?

a) 5% b) 8% c) 10% d) 12%

15. The effective rate which is equivalent to nominal rate of 10% per annum compounded quarterly is

a) 10.25% b) 10.38% c) 10.47% d) 10.53%

16. The maximum value of a function $Z = 7x + 5y$, subject to constraints $x \leq 3, y \leq 2, x \geq 0, y \geq 0$ is

a) 21 b) 10 c) 31 d) 37

17. Region represented by $x \geq 0, y \geq 0$ lies in

a) I quadrant b) II quadrant c) III quadrant d) IV quadrant

18. The straight line trend is represented by

a) $y = a + bx$ b) $y = a - \frac{b}{x}$ c) $y = na + bx$ d) $y = na - bx$

Assertion – Reason Based Questions

The following questions consists of two statements – Assertion (A) and Reason (R). Answer the question selecting appropriate option given below:

a) Both A and R are true and R is correct explanation for R.
b) Both A and R are true but R is not correct explanation for R.
c) A is true but R is false.

d) A is false but R is true.

19. **Assertion (A)** : The differential equation represents the family of parabolas $y^2 = 4ax$, is $x\frac{dy}{dx} - 2y = 0$, where $'a'$ is the parameter.
Reason (R) : If the given family of curves has n parameters then it is to be differentiated n times to eliminate the parameter and obtain the n^{th} order differential equation.

20. **Assertion (A) :** The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ -1 & 2 & x \end{bmatrix}$ is singular for $x = 5$
Reason (R) : A square matrix A is singular if $|A| = 0$

Section – B

21. Evaluate : $\int x\ e^{3x}\ dx$

22. A machine costing Rs. 2,00,000 has effective life of 7 years and its scrap value is Rs. 30,000. What amount should the company put into a sinking fund earning 5% p.a. so that it can replace the machine after its useful life? Assume the new machine will cost Rs. 3,00,000 after 7 years. $[(1.05)^7 = 1.407]$

23. Suppose 2% of the items made by a factory are defective. Find the probability that there are 3 defective items in a sample of 100 times. (given $e^{-2} = 0.135$)

24. Construct a 2×3 matrix $A = [aij]$ whose elements are given by $aij = \frac{i-j}{i+j}$

25. A man takes twice as long as to row a distance against the stream as to row the same distance in the direction of stream. Find ratio of speed of man in still water to the speed of stream.

Section – C

26. Solve $\frac{2x-3}{4} + 9 \geq 3 + \frac{4x}{3}$

27. Solve the following system of equation, using Cramer's rule

$$x + 2y + z = 7, x + 3z = 11, 2x - 3y = 1$$

28. Vikas invested Rs. 10,000 in a stock of a company for 6 years. The value of his investment at the end of each year is given below:

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Rs. 11,000	Rs. 11,500	Rs. 11,650	Rs. 11,800	Rs. 12,200	Rs. 14,000

Using formula, calculate CAGR of his investment (Use $(1.4)^{1/6} = 1.058$)

29. Samples of two types of electric light bulbs were tested for length of life and following data were obtained:

	Type I	Type II
Sample size	$n_1 = 8$	$n_2 = 7$
Sample means	$\overline{X_1} = 1234\ hrs$	$\overline{X_2} = 1036\ hrs$

3-

Sample S.D's $S_1 = 36\ hrs$ $S_2 = 40\ hrs$

If the difference in the means sufficient to warrant that type I is superior to type II regarding length of life? (Use t_{13} (0.05)=2.216)

30. An unbiased die is thrown again and again until three sixes are obtained. Find the probability of obtaining 3rd six in the sixth throw of the die.

31. Determine the intervals in which the function $f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$ is decreasing or increasing.

Section - D

32. Fit a straight line trend by method of least squares and find the trend value for following:

Year	2001	2002	2003	2004	2005	2006	2007
Products	30	35	36	32	37	40	36

33. Avni takes a loan of Rs. 5,00,000 from a bank at an interest rate of 6% p.a. for 10 years. She wants to pay back the loan in EMI's. Find her EMI by using

    a) Flat Rate Method b) Reduced balance method $[(1.005)^{-120} = 05496)$

34. Three pipes A, B and C when opened together can fill a tank in 3/2 hrs. Pipes B and C together take 2 hrs to fill the tank while pipes A and C together take 3 hrs to fill the tank. How long will the pipes A and B together take to fill the tank completely?

35. a) Evaluate $\int_1^3 \frac{1}{x^2(x+1)}\,dx$

    b) $y\,dx - (x + 2y^2)\,dy = 0$

Section - E (Case Studies)

36. In number theory, it is often important to find factors of an integer N. The number N has two trivial factors, namely I and N. Any other factor, if exists, is called non-trivial factor of N. Naresh has plotted a graph of some constraints (linear inequations) with points A (0, 50), B (20, 40), c (50, 100), D (0, 200) and E (100, 0). This graph is constructed using three non-trivial constraints and two trivial constraints. One of the non-trivial constraints is $x + 2y \geq 100$.

Based on the above information, answer the following questions:

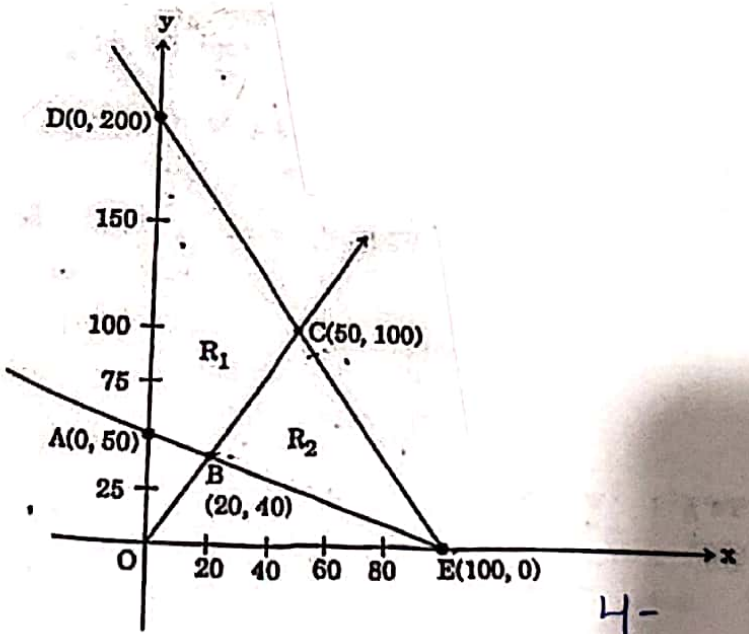
a) What are the two trivial constraints?

b) i) If R_1 is the feasible region, then what are the other two non-trivial constraints?

OR

ii) If R_2 is the feasible region, then what are the other two non-trivial constraints?

c) If R_1 is the feasible region, then find the maximum value of the objective function $z = 5x + 2y$.



37. Consider the differential equation of the form

The marginal revenue of a product (in Rs.) is given by $MR = 7 - 4x + 3x^2$, where x is the number of units produced and sold.

Based on above information, answer the following questions:

a) Find the revenue function.

OR

If at $x = 4$ revenue generated is Rs. 20, then find the revenue function.

b) Find the demand function.

c) Find the average revenue.

38. Amit, Biraj and Chirag were given the task of creating a square matrix of order 2.

Below are the matrices created by them. A, B, C are the matrices created by Amit, Biraj and Chirag respectively.

$A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$

Based on the above information, answer the following questions

a) Find the sum of the matrices A, B and C

b) Find $(A^T)^T$

c) Find $AC - BC$

5-